Conditional relaxation of a charge state under continuous weak measurement

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We investigate the conditional evolution of a charge state coupled to a mesoscopic detector under continuous weak measurement. The state suffers relaxation into a particular state with a definite charge when electrons in a particular output lead are monitored in the detector. The process of the conditional relaxation is not restricted by the shot noise of the detector, unlike the case of the back-action dephasing. As a result, the relaxation of conditional evolution is much faster than the current-sensitive part of dephasing. Furthermore, the direction of the relaxation depends on the choice of the output lead. We propose that these properties can be verified in a two-path interferometer containing a quantum dot capacitively coupled to a detector. In this setup, the current-current correlation between the interferometer and the detector reveals characteristic features of conditional relaxation.

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I. INTRODUCTION

The quantum measurement problem continues to attract interest because a measurement process inevitably causes the "wave-function reduction" that cannot be described in terms of the Schrödinger equation.[1] Mesoscopic physics has recently progressed into a stage that enables us to treat this issue. In particular, a quantum dot entangled with a mesoscopic conductor undergoes "back-action dephasing" which has been experimentally verified.[2–4] This dephasing has also been a subject of intensive theoretical investigation.[5–17] The back-action dephasing can be understood in terms of the possibility of acquiring charge-state information. However, it is important to note that the actual measurement has not been performed for the dephasing process. It only refers to the possibility of measurement and is a result of averaging over all possible measurement outcomes. On the other hand, a quantum measurement performed on the detector brings about a sudden reduction in the charge state (or the "wave-function collapse").[18] Continuous measurement on a particular outcome of the detector state results in an evolution of the charge state in a way that depends on the choice of measurement outcome.

The system under study is schematically drawn in Fig. 1. A quantum point-contact (QPC) adjacent to a charge qubit (usually a double quantum dot) can be used as a charge detector through the charge sensitivity of the detector current.[2,4,19] The information of the charge state is transferred to the detector in the form of a quantum entanglement. There are two possible outcomes of measurement in the QPC detector, that is, transmission and reflection, for each of the detector electrons. Transport through a quantum dot coupled to a QPC detector depends on what detector output current is observed,[20] demonstrating the conditional statistics. The nature of electron transport in the detector is stochastic because of random partitioning at the QPC. The stochastic evolution of the charge state under this random selection of the detector state has been studied before.[21,22]

In our study, in contrast, we investigate the evolution of the state of the charge qubit with the condition that only one particular lead of the detector is intentionally monitored. Our main observations are: (1) the initial state given as a coherent superposition of two different charge states is relaxed to the one of the fixed charge state. The direction of the relaxation depends on the choice of measurement on the detector. That is, the charge state is relaxed to $|0\rangle$ (state without an extra charge on the quantum dot adjacent to the QPC) conditioned on the selection of the detector electron at T. On the other hand, the charge state is relaxed to $|1\rangle$ (state with an extra charge) when electrons are continuously selected at lead R. (2) The relaxation rate is the same in both cases and is proportional to the charge sensitivity of the detector transmission. The relaxation rate is much larger than the current-sensitive part of the dephasing rate, which can be regarded as a manifestation of nonlocality in a measurement process.

We propose an experimental setup which can be used to verify this conditional relaxation. In order to monitor the state of the target system, we introduce a quantum dot embedded in a two path interferometer. The electronic Mach-Zehnder interferometer with a quantum Hall edge channel[23] is an ideal system for this purpose, but the conventional type of Aharonov-Bohm interferometer[24] can also be used. For charge detection, a QPC is considered which is capacitively coupled to the quantum dot. We show that, while the current oscillation amplitude in the interferometer is directly related to dephasing via entanglement, the cross correlation of the currents (between a lead of the interferometer and the other in the detector) reveals the characteristic features of the conditional relaxation. This is possible through conditional statistical average of the state of the interferometer.

The paper is organized as follows. In Sec. II, we formulate the problem of a target state coupled to a mesoscopic detector, and briefly review on the dephasing of the target state as a result of entanglement. In Sec. III, we discuss our main idea of the conditional relaxation under continuous weak measurement of a particular output of the detector. Section IV is devoted to the discussion on the possible experimental verification of the conditional relaxation by constructing a two-path interferometer. The conclusion will be given in Sec. V

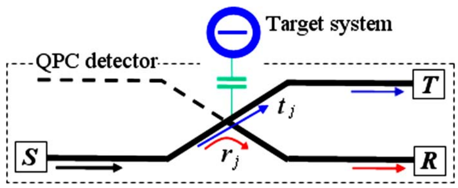

FIG. 1. (Color online) A schematic of a target state coupled to a quantum point-contact detector. The state information is encoded in the charge-dependent reflection and transmission amplitudes, r_j and t_j, respectively, in the detector ($j=0,1$).

II. FORMULATION AND DEPHASING OF THE CHARGE STATE

Initially, the charge state of the target system is in general given as a linear superposition, $a_0|0\rangle+a_1|1\rangle$, of two different charge states, $|0\rangle$ and $|1\rangle$, respectively. Electron scattering via QPC detector is affected by the state of the target system and is accounted for in the scattering matrix (for $j=0,1$)

$$S_{pc}=(\delta_{j0}+\delta_{j1})\begin{pmatrix} r_j & t_j' \\ t_j & r_j' \end{pmatrix}, \tag{1}$$

where its elements depend on the charge state $|j\rangle$. For a detector bias $V_{\rm det}$, the average number of electrons injected into the QPC during the time interval t is $n=eV_{\rm det}t/h$. We are interested in the limit of continuous measurement, that is, $n\gg 1$, and neglect the energy dependence of the matrix elements.

The electron creation(annihilation) of energy ϵ at lead l ($l=S,T,R$) is represented by the operator $c_l^\dagger(\epsilon)[c_l(\epsilon)]$. The initial state is a direct product of the charge state $a_0|0\rangle+a_1|1\rangle$ and the detector state $\Pi_{0<\epsilon\le eV_{\rm det}}c_S^\dagger(\epsilon)|F\rangle$ where $|F\rangle$ is the Fermi sea of electrons with their energies lower than zero. Upon interaction of n detector electrons with the charge state, the two subsystems get entangled as

$$|\Psi\rangle=a_0|0\rangle\otimes\left[\prod_\epsilon\chi_0^\dagger(\epsilon)\right]|F\rangle+a_1|1\rangle\otimes\left[\prod_\epsilon\chi_1^\dagger(\epsilon)\right]|F\rangle, \tag{2a}$$

where the energy interval $0\le\epsilon\le eV_{\rm det}$ is counted. $\chi_j^\dagger(\epsilon)$ ($j=0,1$) creates a charge-state-dependent detector electron,

$$\chi_j^\dagger(\epsilon)=r_jc_R^\dagger(\epsilon)+t_jc_T^\dagger(\epsilon). \tag{2b}$$

Dephasing of the charge state induced by this type of entanglement is now well understood.[5–15] First, we briefly review the dephasing properties of the charge state. The charge state is described by a reduced density matrix $\rho={\rm Tr}_{\rm det}[|\Psi\rangle\langle\Psi|]$, where ${\rm Tr}_{\rm det}[\cdots]$ sums over the detector's degrees of freedom. From this, we can find the time evolution of the density-matrix elements,

$$\ln\rho_{jj'}(t)=\ln\rho_{jj'}(0)+\sum_{0<\epsilon\le eV_{\rm det}}\ln[\nu_{jj'}(\epsilon)], \tag{3}$$

where $\nu_{jj'}(\epsilon)=r_{j'}^*r_j+t_{j'}^*t_j$ is the quantity that accounts for the effect of charge detection. The initial density matrix is $\rho_{jj'}(0)=a_ja_{j'}^*$. Equation (3) indicates that the diagonal components are unchanged, but the off-diagonal terms decay as a function of time leading to dephasing. In the limit of $t\gg h/eV_{\rm det}$ with $|\nu_{01}(\epsilon)|\sim 1$ (weak continuous measurement), we obtain the asymptotic relation $|\rho_{01}(t)|=|\rho_{01}(0)|\exp(-\Gamma_{\rm dep}t)$ where the dephasing rate $\Gamma_{\rm dep}$ is given by $\Gamma_{\rm dep}=-\int h^{-1}d\epsilon\ln|\nu_{01}(\epsilon)|$. Due to the condition of weak measurement ($|\nu_{01}(\epsilon)|\sim 1$), $\Gamma_{\rm dep}$ can be expanded in terms of the change in the transmission probability

$$\Delta\mathcal{T}=\mathcal{T}_0-\mathcal{T}_1$$

($\mathcal{T}_j\equiv|t_j|^2$) and the change in the relative scattering phase $\Delta\phi\equiv\arg(t_0/r_0)-\arg(t_1/r_1)$. We find

$$\Gamma_{\rm dep}=\frac{eV_{\rm det}}{8h}\frac{(\Delta\mathcal{T})^2}{\mathcal{T}(1-\mathcal{T})}+\frac{eV_{\rm det}}{2h}\mathcal{T}(1-\mathcal{T})(\Delta\phi)^2, \tag{4}$$

where $\mathcal{T}=(\mathcal{T}_0+\mathcal{T}_1)/2$.

III. CONDITIONAL RELAXATION OF THE CHARGE STATE

Next, we discuss our main observation of the conditional evolution of the charge state. In the above, we have described dephasing of the charge state by its entanglement with the detector electrons. Actual measurement for the detector is not performed for dephasing of the charge state. In contrast, we can monitor the charge state of the target system under continuous selection of detector electrons at a particular lead. (This corresponds to a continuous projective measurement.) The conditional state is obtained by projecting the total state into a state with a specific outcome of measurement and renormalizing the reduced wave function.[18] It is important to note that, under this circumstance, the charge state is not entangled with the detector state, and remains as a pure state. In the particular setup of Fig. 1, there are two possible outcomes for measurement on the detector, that is, transmission and reflection, for each of the detector electrons. So, there are two different ways of continuous projection for the detector outputs. This measurement is given by the operator

$$M_y=N_y|y\rangle\langle y|, \tag{5a}$$

where $|y\rangle=[\Pi_\epsilon c_y^\dagger(\epsilon)]|F\rangle$ and $N_y=[\langle\Psi|y\rangle\langle y|\Psi\rangle]^{-1/2}$. The case $y=R$ ($y=T$) corresponds to a continuous projection of the detector state onto lead $R(T)$. The corresponding state of the composite system evolves as

$$|\Psi\rangle\to M_y|\Psi\rangle=|\psi^y(t)\rangle\otimes|y\rangle, \tag{5b}$$

where

$$M_R|\Psi\rangle=N_R\left[a_0\left(\prod_\epsilon r_0\right)|0\rangle+a_1\left(\prod_\epsilon r_1\right)|1\rangle\right]\otimes|R\rangle,$$

$$M_T|\Psi\rangle=N_T\left[a_0\left(\prod_\epsilon t_0\right)|0\rangle+a_1\left(\prod_\epsilon t_1\right)|1\rangle\right]\otimes|T\rangle.$$

Clearly, the two subsystems are disentangled upon the measurement as a result of the "wave-function collapse." From the Eq. (5b), the conditional state of the target system is given by

$$|\psi^y(t)\rangle = A_y(t)|0\rangle + B_y(t)|1\rangle, \tag{5c}$$

where the coefficients $A_y(t)$ and $B_y(t)$ satisfy the relations,

$$\frac{B_R(t)}{A_R(t)} = \frac{a_1}{a_0}\prod_{\epsilon}\frac{r_1}{r_0}, \quad \frac{B_T(t)}{A_T(t)} = \frac{a_1}{a_0}\prod_{\epsilon}\frac{t_1}{t_0}. \tag{5d}$$

In the asymptotic limit ($t \gg h/(eV_{\rm det}\mathcal{R}_0)$ for $y=R$, $t \gg h/(eV_{\rm det}\mathcal{T}_0)$ for $y=T$), we find

$$\frac{B_R(t)}{A_R(t)} = e^{(\Gamma_{\rm rel}^R/2+i\xi_R)t}\frac{a_1}{a_0}, \quad \frac{B_T(t)}{A_T(t)} = e^{(-\Gamma_{\rm rel}^T/2+i\xi_T)t}\frac{a_1}{a_0}, \tag{5e}$$

where the relaxation rates are

$$\Gamma_{\rm rel}^R = 2\mathcal{R}_0\int h^{-1}d\epsilon \ln|r_1/r_0|, \tag{5f}$$

$$\Gamma_{\rm rel}^T = -2\mathcal{T}_0\int h^{-1}d\epsilon \ln|t_1/t_0|. \tag{5g}$$

Here $\mathcal{R}_j=|r_j|^2$ is the reflection probability. The measurement also induces the phase shifts $\xi_R=\mathcal{R}_0 eV_{\rm det}\arg(r_1/r_0)/h$ and $\xi_T=\mathcal{T}_0 eV_{\rm det}\arg(t_1/t_0)/h$, respectively. Imposing conditions for weak measurement, ($\Delta\mathcal{T}/\mathcal{R}_0 \ll 1$ for $y=R$ and $\Delta\mathcal{T}/\mathcal{T}_0 \ll 1$ for $y=T$), we find that $\Gamma_{\rm rel}^R=\Gamma_{\rm rel}^T=\Gamma_{\rm rel}$, where

$$\Gamma_{\rm rel} = \frac{eV_{\rm det}}{h}\Delta\mathcal{T}. \tag{5h}$$

With some algebra, we obtain $A_y(t)=N_y e^{u_y t}a_0$ and $B_y(t)=N_y e^{v_y t}a_1$, where

$$u_R = \mathcal{R}_0 eV_{\rm det}\ln(r_0)/h,$$

$$v_R = \mathcal{R}_1 eV_{\rm det}\ln(r_1)/h,$$

$$u_T = \mathcal{T}_0 eV_{\rm det}\ln(t_0)/h,$$

and

$$v_T = \mathcal{T}_1 eV_{\rm det}\ln(t_1)/h.$$

Implications of these results [Eq. (5)] are summarized as follows. First, the charge state evolves into $|0\rangle(|1\rangle)$ with the relaxation rate $\Gamma_{\rm rel}$ [Eq. (5h)] under continuous projection of detector electrons onto lead $T(R)$. The direction of the evolution depends on which output lead is selected. It is important to note that the conditional state remains as a pure state as a result of measurement, in contrast to the case of dephasing. We also point out that the conditional relaxation considered here is different from the stochastic evolution under random selection of measurement outcome due to the partition noise of the QPC.[21,22] In the stochastic evolution, the evolution of the target state is affected both by the transmitted and the reflected electrons. On the other hand, in the conditional evolution described by [Eq. (5)], it does not matter that there are other electrons not monitored in the QPC. In the measurement [Eq. (5)], we monitor only electrons of lead y. In other words, the information obtained by the measurement is selective. Under this measurement, the state of charge evolves following Eq. (5c), and this relaxation can be observed in correlation experiment. In order to observe conditional relaxation under monitoring only one particular output lead, we need to correlate the state of the target system with that of the detector output. (See below for observing this correlation.) Second, $\Gamma_{\rm rel}$ is much larger than $\Delta\mathcal{T}$-dependence $\Gamma_{\rm dep}$. Because only one particular output is continuously selected, the conditional relaxation is not restricted by the shot noise of the QPC detector, unlike the dephasing process. Finally, $\Gamma_{\rm rel}$ depends only on $\Delta\mathcal{T}$, while $\Gamma_{\rm dep}$ depends both on $\Delta\mathcal{T}$ and $\Delta\phi$. Dephasing is related to the state information transferred to the detector and therefore to the possibility of measurement. On the other hand, by selecting one particular lead in the detector, the phase part ($\Delta\phi$) of the state information is erased. In fact, this behavior is equivalent to the quantum erasure of the charge-state information encoded in the relative scattering phase $\Delta\phi$.[25]

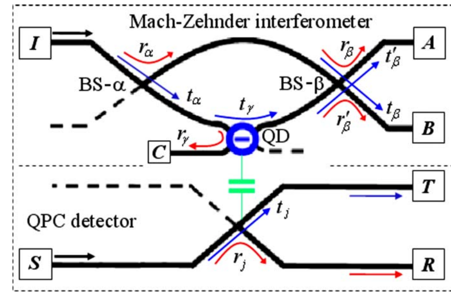

FIG. 2. (Color online) A schematic of a two-path interferometer coupled to a quantum point-contact detector. Cross-correlation measurement between an output lead from the interferometer (A or B) and the other from the detector (T or R) reveals the nature of conditional relaxation. (See text for a discussion.)

IV. TWO-PATH INTERFEROMETER COUPLED TO A QUANTUM POINT CONTACT DETECTOR

Next, we propose a possible experiment to observe the effect of the conditional relaxation. For a target system, we consider an electronic two-path interferometer with a quantum dot (QD) embedded in one of the two paths. The QD is capacitively coupled to a QPC detector (see Fig. 2). In fact, the two-path interferometer can be regarded as a charge qubit aside from the scattering at the QD. We simply discard the reflection at the QD in our formulation. The two-path interferometer can be implemented by constructing a double-slit-type Aharonov-Bohm interferometer.[24] Alternatively, it can be built up by two beam splitters (BS-α and BS-β) with quantum Hall edge state. This is an electronic analog of the Mach-Zehnder interferometer (MZI).[23] The electronic transport in the interferometer is characterized by the scattering matrix at BS-α, BS-β and QD,

$$S_i = \begin{pmatrix} r_i & t_i' \\ t_i & r_i' \end{pmatrix}, \tag{6}$$

where $i=\alpha,\beta,\gamma$. The reflection and the transmission probabilities are written as $\mathcal{R}_i=|r_i|^2$ and $\mathcal{T}_i=|t_i|^2$, respectively.

Because of the dwell time in the QD (denoted as Γ^{-1}), the dephasing effect due to coupling to the QPC detector appears in the probability (P_x) to find an electron at lead x ($x=A,B$),

$$P_x = \mathrm{Tr}_{\mathrm{MZI}}[c_x^\dagger c_x \rho(t=\Gamma^{-1})], \tag{7a}$$

where $\mathrm{Tr}_{\mathrm{MZI}}[\cdots]$ sums over the MZI degree of freedom. Equation (7a) implies that the electron is (on average) collected at lead x after time $t=1/\Gamma$ upon injection. it gives

$$P_A = \mathcal{R}_\alpha \mathcal{R}_\beta + \mathcal{T}_\alpha \mathcal{T}_\beta \mathcal{T}_\gamma + 2\mathcal{V}M \cos(\varphi + \phi_\nu), \tag{7b}$$

$$P_B = \mathcal{R}_\alpha \mathcal{T}_\beta + \mathcal{T}_\alpha \mathcal{R}_\beta \mathcal{T}_\gamma - 2\mathcal{V}M \cos(\varphi + \phi_\nu), \tag{7c}$$

where $\varphi=\arg(t_\alpha t_\gamma t'_\beta / r_\alpha r_\beta)$, $M=(\mathcal{R}_\alpha \mathcal{R}_\beta \mathcal{T}_\alpha \mathcal{T}_\beta \mathcal{T}_\gamma)^{1/2}$, and $\phi_\nu = eV_{\mathrm{det}} \arg(\nu_{01})/h\Gamma$. The visibility factor ($\mathcal{V}$) depends on the dephasing rate of Eq. (4) as

$$\mathcal{V} = e^{-\Gamma_{\mathrm{dep}}/\Gamma} \simeq 1 - \Gamma_{\mathrm{dep}}/\Gamma, \tag{7d}$$

in the limit of $\Gamma_{\mathrm{dep}}/\Gamma \ll 1$, which agrees with previous results.[5,12]

The conditional probability ($P_{x|y}$) to find an electron at lead x ($x=A,B$) conditioned on a particular detector output y ($y=T,R$) is obtained from the relation $P_{x|y} = \langle \Psi | M_y c_x^\dagger c_x M_y | \Psi \rangle$. At time $t=1/\Gamma$, it is given as

$$P_{A|y} = N_y^2[\mathcal{R}_\alpha \mathcal{R}_\beta e^{2\,\mathrm{Re}(u_y)/\Gamma} + \mathcal{T}_\alpha \mathcal{T}_\beta \mathcal{T}_\gamma e^{2\,\mathrm{Re}(v_y)/\Gamma} + 2Me^{\mathrm{Re}(u_y+v_y)/\Gamma} \cos(\varphi + \xi_y/\Gamma)], \tag{8a}$$

$$P_{B|y} = N_y^2[\mathcal{R}_\alpha \mathcal{T}_\beta e^{2\,\mathrm{Re}(u_y)/\Gamma} + \mathcal{T}_\alpha \mathcal{R}_\beta \mathcal{T}_\gamma e^{2\,\mathrm{Re}(v_y)/\Gamma} - 2Me^{\mathrm{Re}(u_y+v_y)/\Gamma} \cos(\varphi + \xi_y/\Gamma)], \tag{8b}$$

where $N_y=[e^{2\,\mathrm{Re}(u_y)/\Gamma}\mathcal{R}_\alpha + e^{2\,\mathrm{Re}(v_y)/\Gamma}\mathcal{T}_\alpha]^{-1/2}$ ($y=R,T$). The amplitude of the interference term in $P_{x|y}$ is modulated by the visibility factor $\mathcal{V}_y$,

$$\mathcal{V}_y = N_y^2 e^{\mathrm{Re}(u_y+v_y)/\Gamma} \cong 1 \pm \left(\frac{1}{2} - \mathcal{T}_\alpha\right)\frac{\Gamma_{\mathrm{rel}}}{\Gamma}, \tag{8c}$$

where $+(-)$ sign is for $y=R(T)$. In contrast to the case of dephasing in single-particle transport, the visibility of the conditional probability can be enhanced or reduced depending on the transmission probability $\mathcal{T}_\alpha$ of the BS-α in the MZI and the choice of the output lead in the detector. When $\mathcal{T}_\alpha < 1/2$ ($\mathcal{T}_\alpha > 1/2$), the visibility is enhanced for $y=R$ ($y=T$) and reduced for $y=T$ ($y=R$). This behavior of the visibility can be understood by considering the direction of the conditional relaxation in the Bloch sphere (Fig. 3). The amplitude of the interference term is proportional to the radius of the circle of its latitude. The amplitude is maximum at the equator ($\mathcal{T}_\alpha=1/2$) which corresponds to the symmetric beam splitting. Therefore, the interference signal is enhanced if the conditional state evolves toward equator of the Bloch sphere while it is reduced if the state evolves toward two poles. This explains the visibility change of Eq. (8c) in the limit of weak measurement.

In the following, we show that the cross-correlation measurement of current at leads x ($x=A,B$) and y ($y=T,R$) is directly related to the quantity $P_{x|y}$ in Eq. (8). The bias voltage, V, applied to the MZI is assumed to be much smaller than that of the detector: $V \ll V_{\mathrm{det}}$. The frequency-dependent current cross correlation $S_{xy}(\omega)$ is defined by

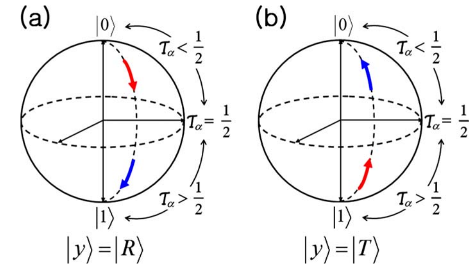

FIG. 3. (Color online) Schematic for describing the conditional relaxation (a) for $|y\rangle=|R\rangle$ and (b) for $|y\rangle=|T\rangle$. The arrows indicate the direction of the conditional evolution induced by the measurement of Eq. (5). For arrows toward equator (poles), the visibility of the conditional probability is enhanced (reduced) as a result of measurement.

$$2\pi\delta(\omega+\omega')S_{xy}(\omega) = \langle \bar{\Psi} | \Delta\mathcal{I}_x(\omega)\Delta\mathcal{I}_y(\omega') + \Delta\mathcal{I}_y(\omega')\Delta\mathcal{I}_x(\omega) | \bar{\Psi} \rangle, \tag{9}$$

where $|\bar{\Psi}\rangle$ is the many-electron transport state of the composite system. $\Delta\mathcal{I}_l$ is the current fluctuation defined by $\Delta\mathcal{I}_l = \mathcal{I}_l - \langle \mathcal{I}_l \rangle$ where $\mathcal{I}_l$ is the output current operator at lead l.

In evaluating the expectation values in Eq. (9), we need to calculate quantities such as $\langle \bar{\Psi} | c_x^\dagger(E) c_x(E') c_y^\dagger(\epsilon) c_y(\epsilon') | \bar{\Psi} \rangle$. E,E' and ϵ,ϵ' are the energies of electrons injected from the interferometer and the detector, respectively. These energies are in the ranges $0 \le E$, $E' \le eV$ and $0 \le \epsilon$, $\epsilon' \le eV_{\mathrm{det}}$. In order to calculate such quantities, we made the following assumptions: (i) all of the scattering matrices are independent of the energies. This assumption is valid as long as the bias voltages are not very large to alter the characteristics of the QPCs. (ii) The density matrix of the whole system, $\bar{\rho} \equiv |\bar{\Psi}\rangle\langle\bar{\Psi}|$ can be written as a direct product,

$$\bar{\rho} \simeq \bar{\rho}_1 \otimes \bar{\rho}_2, \tag{10}$$

where $\bar{\rho}_1$ is the part of the density matrix that contains energies E,E', ϵ,ϵ', while $\bar{\rho}_2$ represents the remaining part. This is a reasonable assumption because the different energy states of electrons are unlikely to interfere with each other. Using these assumptions, we obtain a simple relation of the zero-frequency cross correlation,

$$S_{xy}(0) = \frac{e^3}{\pi\hbar} V[P_{x|y}P_y - P_x P_y], \tag{11}$$

where $P_R = \mathcal{R}_\alpha \mathcal{R}_0 + \mathcal{T}_\alpha \mathcal{R}_1$ and $P_T = 1 - P_R$. Also, it is straightforward to find that the average current $\langle \mathcal{I}_x \rangle$ at lead x satisfies the Landauer formula: $\langle \mathcal{I}_x \rangle = (e^2/2\pi\hbar)P_x V$. (Similarly, $\langle \mathcal{I}_y \rangle = (e^2/2\pi\hbar)P_y V_{\mathrm{det}}$ for the detector.) Therefore, analyzing the cross correlation $S_{xy}(0)$ as well as the dc currents reveals the characteristic features of conditional relaxation and dephasing.

V. CONCLUSION

In conclusion, we have found that a linearly superposed charge state is conditionally relaxed under continuous measurement by an attached QPC detector. The direction of the relaxation depends on the choice of the detector output lead. It takes place much faster than the current-sensitive part of dephasing. We suggest that this feature can be revealed by constructing an interferometer for the charge state and investigating the current-current correlation between the two subsystems.

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